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ATTENUATE: A SAS® Macro for Computing Confidence Intervals for Disattenuated Correlation Coefficients

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ABSTRACT

Correcting correlation coefficients for attenuation resulting from measurement error has a long history (dating back to Spearman's work in 1904). However, the estimation of confidence intervals for disattenuated correlation coefficients is a more recent concern. The macro described in this paper computes the point estimate of the correlation coefficient after adjusting for measurement error, and the endpoints of two estimates of confidence intervals: the traditional Fisher/Spearman interval and the confidence set suggested by Charles (2005). The macro is written in Base SAS software, with arguments consisting of the obtained sample correlation coefficient, estimates of the reliability of each variable, sample size, and level of confidence for the intervals. Through providing a convenient method to compute disattenuated correlation coefficients and accurate confidence intervals, this paper will encourage researchers to be mindful of the impact of measurement error on indices of correlation, as well as the consideration of sampling error in the interpretation of results.

Keywords: MEASUREMENT ERROR, STATISTICAL ADJUSTMENT, BASE SAS

INTRODUCTION

Indices of correlation are used to describe the direction and strength of the relationship between two variables. The most frequently used index of correlation is the Pearson Product-Moment Correlation Coefficient:

$$r_{XY} = \frac{\hat{\sigma}_{XY}}{\hat{\sigma}_X \hat{\sigma}_Y}$$

where $\hat{\sigma}_{XY}$ is the sample covariance between the two variables (*X* and *Y*) and $\hat{\sigma}_i$ is the standard deviation of the i^{th} variable.

This coefficient ranges from -1.0 to +1.0, and takes the value of zero when no linear relationship between *X* and *Y* is present in the sample. The Pearson Product-Moment Correlation Coefficient has proven useful as a simple summary statistic for describing bivariate relationships, but it also serves as an important building block for more advanced statistical analyses such as multiple regression analysis and factor analysis.

Inferences about the direction and strength of relationships in populations, rather than samples, are frequently made using both null hypothesis testing (i.e., testing the null hypothesis that the population coefficient of correlation, ρ_{XY}, is zero) and confidence interval estimation. In both forms of inferential statistics, the sample correlation coefficient is used as a point estimate of the population coefficient, that is:

$$r_{XY} = \hat{\rho}_{XY}$$

A variety of research artifacts are known to weaken or attenuate the apparent correlation between variables, including artificial restriction of variability, nonlinearity, and measurement error in the variables. Adjustments for such attenuation are needed to reduce or remove the statistical bias in the coefficient. The SAS macro presented in this paper provides an easy method for disattenuating sample coefficients calculated from variables that are measured with random error.

IMPACT OF MEASUREMENT ERROR ON CORRELATION

Measurement error has been identified as an important factor in the attenuation of correlation for many years. The classic paper by Spearman (1904) brought attention to the phenomenon. True-score theory posits that the observed values of variables (X_i) are functions of the true values of these variables (T_i) and errors of measurement (E_i):

$$X_i = T_i + E_i$$

Under the typical assumptions of true-score theory (that errors of measurement are uncorrelated with true scores and uncorrelated with errors of measurement in other variables), measurement error in variables leads to attenuation of the apparent correlation between the variables, leading to observed correlations that are systematically smaller than the correlations between the true scores. The amount of measurement error in variables is usually represented by an index of reliability:

$$\rho_{XX} = \frac{\sigma_T^2}{\sigma_T^2 + \sigma_E^2}$$

where σ_T^2 is the variance of the true scores and σ_E^2 is the variance of the measurement errors.

The index of reliability is the proportion of observed score variance that is variance in true scores rather than measurement error. The index ranges from zero to one, with higher values indicating measures containing less measurement error.

CORRECTION FOR ATTENUATION AND ESTIMATION OF CONFIDENCE INTERVALS

Spearman's (1904) correction for attenuation derives directly from true-score theory:

$$\rho_{XY}^* = \frac{\rho_{XY}}{\sqrt{\rho_{XX}}\sqrt{\rho_{YY}}}$$

where ρ_{XY}^* is the corrected, or disattenuated, correlation (that is, the correlation between the true scores), ρ_{XY} is the correlation between the observed scores, and the ρ_{ii} are the reliabilities of the two variables.

Although Spearman's correction for attenuation provides a reasonable point estimate for the correlation between two variables after adjusting for errors of measurement, the construction of confidence intervals around this correlation has been more difficult. The traditional approach to confidence interval estimation (e.g., Hunter & Schmidt, 1990; Winne & Belfrey, 1982) uses the Fisher (1950) normalization of the obtained correlation coefficient to construct a confidence interval around the uncorrected coefficient, then disattenuates the endpoints of the resulting interval. Thus, for a 95% confidence interval around an observed sample correlation $\hat{\rho}_{XY}$:

$$Z_{\hat{\rho}} = \frac{1}{2}\ln\left(\frac{1+|\hat{\rho}_{XY}|}{1-|\hat{\rho}_{XY}|}\right)$$

$$Z_{\hat{\rho}} - \frac{1.96}{\sqrt{N-3}} < Z_{\rho} < Z_{\hat{\rho}} + \frac{1.96}{\sqrt{N-3}}$$

The endpoints of this interval are then back-transformed to the correlation metric:

$$\frac{e^{2Z_{\hat{\rho}}+\frac{1.96}{\sqrt{N-3}}-1}}{e^{2Z_{\hat{\rho}}+\frac{1.96}{\sqrt{N-3}}+1}} < \rho_{XY} < \frac{e^{2Z_{\hat{\rho}}-\frac{1.96}{\sqrt{N-3}}-1}}{e^{2Z_{\hat{\rho}}-\frac{1.96}{\sqrt{N-3}}+1}}$$

In the final step, the endpoints are disattenuated using Spearman's (1904) correction for attenuation.

An alternative approach to interval estimation was suggested by Charles (2005). Using extensive simulations, Charles derived upper and lower limits to the intervals as:

$$Upper = \frac{-1 + \sqrt{1 - 4(-1.96M)(-1.96L - \hat{\rho}^*_{XY})}}{2(-1.96M)}$$

$$Lower = \frac{-1 + \sqrt{1 - 4(1.96M)(1.96L - \hat{\rho}^*_{XY})}}{2(1.96M)}$$

$$where\ \ L = -.0608\left(\frac{1}{\sqrt{N}}\right) - .00455\left(\frac{1}{\bar{\rho}_{ii}}\right) + 1.11\left(\frac{1}{\sqrt{N}}\frac{1}{\bar{\rho}_{ii}}\right)$$

$$and\ \ M = .00759 - 1.12\left(\frac{1}{\sqrt{N}}\right) - .00355\left(\frac{1}{\bar{\rho}_{ii}}\right) + .152\left(\frac{1}{\sqrt{N}}\frac{1}{\bar{\rho}_{ii}}\right)$$

MACRO ATTENUATE

A SAS macro was designed to compute the disattenuated point estimate of the correlation, and the endpoints of both methods of interval estimation. The macro was developed to provide researchers with an easily accessible tool for calculating these statistics. Arguments supplied to the macro include the name of the SAS dataset that contains the other arguments, the uncorrected sample correlation coefficient, the sample size, reliability estimates for both variables, and the desired level of confidence for the intervals. In addition, an indicator argument is used to request html output via the SAS Output Delivery System (ODS). By default, the macro uses the latest SAS data set created, variable reliability values of .80, a 95% level of confidence, and no ODS output.

The macro initializes a new SAS data step and performs the relevant computations for the disattenuation and the interval estimation. The output from the macro includes a table to present the calculated statistics, as well as descriptive information about the data analyzed. Of course, the macro syntax may be easily modified to write the output to a disk file or to send the statistics to regular SAS for further analyses.

ATTENUATE Code

```
%macro ATTENUATE (DATASET = _LAST_, N = N, OBS_CORR = rxy, RXX_1 = rxx1, RXX_2 = rxx2,
                      Confidence = .95, WantODS = 0);

* +-------------------------------------------------------------------------------------------+
       Macro Arguments:
              DATASET = SAS Dataset containing summary data on the correlation
              N = Sample size on which correlation was computed
              OBS_CORR = Observed sample Pearson Product-Moment Correlation Coefficient
              RXX_1 = Estimated reliability for variable 1
              RXX_2 = Estimated reliability for variable 2
              Confidence = Level of confidence for interval estimates
              WantODS = Indicator argument to request ODS output as html (set WantODS = 1)
  +-------------------------------------------------------------------------------------------+;

       * +--------------------------------------------+
              Enable html Output Through ODS if Requested
         +--------------------------------------------+;
              %if &wantODS = 1 %then %do;
                     OPTIONS FORMCHAR="|----|+|---+=|-/\<>*";
```

```
                    ods html;
                    ods html body='your-html-file.html';
            %end;

data temp;
 set &dataset;
 CL = &confidence * 100;

  * +--------------------------------------------------+
      Disattenuate the Sample Correlation Coefficient
    +--------------------------------------------------+;
       dis_corr = &obs_corr/(sqrt(&rxx_1)*sqrt(&rxx_2));

  * +-------------------------------------------------------------+
         Compute Mean Reliability Estimate for the Two Variables
    +-------------------------------------------------------------+;
       mn_reliability = (&rxx_1 + &rxx_2) / 2;

 * +-----------------------------------------------------------------------------------+
        Compute Endpoints of Fisher CI Around Observed Correlation (without disattenuation)
   +-----------------------------------------------------------------------------------+;
    critZ = -1*probit((1-&Confidence)/2);
    transfish_z = 0.5*log((1+abs(&obs_corr))/(1-abs(&obs_corr)));
    if &obs_corr < 0 then transfish_z = transfish_z * -1;
       SE_fish_z = sqrt((&N - 3)**-1);
       upperCI = (exp(2*(transfish_z  + critZ*SE_fish_z)) - 1) /
                    (exp(2*(transfish_z  + critZ*SE_fish_z)) + 1);
       lowerCI = (exp(2*(transfish_z  - critZ*SE_fish_z)) - 1) /
                    (exp(2*(transfish_z  - critZ*SE_fish_z)) + 1);

 * +-------------------------------------------------------+
        Disattenuate the Endpoints of the Fisher Interval
   +-------------------------------------------------------+;
       dis_upper = UpperCI/(sqrt(&rxx_1)*sqrt(&rxx_2));
       dis_lower = LowerCI/(sqrt(&rxx_1)*sqrt(&rxx_2));

 * +-----------------------------------------------------------------------+
        Compute the endpoints of the confidence sets suggested by Charles
   +-----------------------------------------------------------------------+;
       L = -.0608*(1/SQRT(&N)) - .00455*(1/mn_reliability) + 1.111*(1/SQRT(&N)*1/mn_reliability);
       M = .00759 - 1.12*(1/SQRT(&N)) - .00355*(1/mn_reliability) +
              .152*(1/SQRT(&N)*1/mn_reliability);

       Charles_low = (-1 + SQRT(1 - 4*(critZ*M)*(critZ*L - dis_corr)))/(2*(critZ*M));
       Charles_high = (-1 + SQRT(1 - 4*(-critZ*M)*(-critZ*L - dis_corr)))/(2*(-critZ*M));

 * +-------------------------------------------+
        Create Macro Output Using File Print
   +-------------------------------------------+;
       file print header = h notitles;
       put @5 CL 2. @7 '%' @17 LowerCI 6.3 @27 UpperCI 6.3 @40 dis_lower 6.3 @50 dis_upper 6.3
              @63 Charles_low 6.3 @74 Charles_high 6.3;
       return;

       h: put
       @1 'Disattenuation of Correlation Coefficients' ///
       @1 'Observed Correlation:' @35 &obs_Corr 6.3 //
       @1 'Reliability Estimates:' @35 &rxx_1 6.3 /
                                                    @35 &rxx_2 6.3 //
       @1 'Sample Size:'                     @35 &N 6.0 //
       @1 'Disattenuated Correlation:' @35 dis_corr 6.3 ////
```

```
        @1 'Interval Estimates:' //
        @1 '               Observed Correlation            Disattenuated Correlation' /
        @1 '               --------------------     ---------------------------------------' /
        @1 '                 Fisher Interval        Fisher Interval        Charles Interval' /
        @1 'Confidence       ---------------        ---------------        ----------------' /
        @1 '  Level          Lower     Upper        Lower     Upper        Lower      Upper' /
        @1 '----------       -----     -----        -----     -----        -----      -----';
run;
        * +---------------------------------------------+
               Close html Output File if it was Opened
          +---------------------------------------------+;
               %if &wantODS = 1 %then %do;
                       ods html close;
               %end;
* +--------------------+
   Delete TEMP Dataset
  +--------------------+;
        proc datasets;
               delete temp;
        run;

%mend ATTENUATE;
```

EXAMPLES OF MACRO ATTENUATE

The easiest way in which the macro ATTENUATE may be used is to simply create a SAS dataset that inputs the sample summary data. The macro is then called, using as arguments the name of the dataset and the names of the relevant variables. Summary data from 100 observations are used to illustrate the macro. These data provided a sample correlation of .70, and the reliability estimates for the two variables were .80 and .90. The observed data are read into the SAS dataset ONE. The call to the macro requests confidence intervals of 90%, 95%, and 99%. For this example, no html output has been requested, so the macro output is written to the output window in SAS.

```
data one;
 input sampsize r reliable1 reliable2 conf;
  cards;
 100 .70 .80 .90 .90
 100 .70 .80 .90 .95
 100 .70 .80 .90 .99
 ;

Title 'Disattenuation of Correlation Coefficients: No HTML Ouput';
 %attenuate (dataset = one, n = sampsize, obs_corr = r, rxx_1 = reliable1, rxx_2 = reliable2,
               confidence = conf);
```

As a second example, summary data from 200 observations provided a sample correlation of .62, with reliability estimates for the two variables of .70 and .80. The observed data are read into the SAS dataset TWO. As with the first example, the call to the macro requests confidence intervals of 90%, 95%, and 99%. For this example, html output has been requested (the WantODS = 1 argument in the macro call).

```
data two;
 input sampsize r reliable1 reliable2 conf;
  cards;
 200 .62 .70 .80 .90
 200 .62 .70 .80 .95
 200 .62 .70 .80 .99
 ;
Title 'Disattenuation of Correlation Coefficients: Requested HTML Ouput';
 %attenuate (dataset = two, n = sampsize, obs_corr = r, rxx_1 = reliable1, rxx_2 = reliable2,
               confidence = conf, WantODS = 1);
run;
```

OUTPUT FROM MACRO ATTENUATE

Table 1 provides the macro output from the first example. The output displays the values of the arguments sent when the macro was invoked (the observed correlation, reliability estimates, and the sample size). The point estimate of the disattenuated correlation coefficient (.825) is provided, followed by interval estimates based on the observed (e.g., uncorrected) correlation coefficient, and interval estimates of the disattenuated correlation using the Fisher/Spearman and the Charles approaches. The macro program allows for the researcher to garner a point estimate of the disattenuated correlation coefficient for any observed value, reliability estimate and sample size.

Although the observed sample correlation is only .70, after adjusting for the measurement error in the two variables (i.e., r_{ii} = .80 and .90 for the two variables) the expected correlation is notably stronger (r = .825). Without adjusting for the attenuation of the correlation, the 95% confidence interval (the second row of the interval estimates part of the printed output) ranges from .584 to .788. In contrast, following the disattenuation the Fisher/Spearman 95% confidence interval ranges from .688 to .929. The Charles approach yields an interval that is very similar to the Fisher/Spearman interval: with the Charles approach, we can say we are 95% confident that the population correlation is between .671 and .911 if the two variables were measured with perfect reliability.

Table 1
Sample Output from Macro.

```
Disattenuation of Correlation Coefficients


Observed Correlation:              0.700

Reliability Estimates:             0.800
                                   0.900

Sample Size:                         100

Disattenuated Correlation:         0.825



Interval Estimates:

               Observed Correlation          Disattenuated Correlation
               --------------------     ---------------------------------------
                 Fisher Interval        Fisher Interval       Charles Interval
Confidence       ---------------        ---------------       ----------------
  Level          Lower     Upper        Lower     Upper       Lower      Upper
----------       -----     -----        -----     -----       -----      -----
    90%          0.605     0.776        0.712     0.914       0.702      0.900
    95%          0.584     0.788        0.688     0.929       0.671      0.911
    99%          0.541     0.811        0.638     0.955       0.603      0.930
```

Table 2 provides the html macro output from the second example. The output displays the values of the arguments sent when the macro was invoked (the observed correlation, reliability estimates, and the sample size). The point estimate of the disattenuated correlation coefficient (.829) is provided, followed by interval estimates based on the observed (e.g., uncorrected) correlation coefficient, and interval estimates of the disattenuated correlation using the Fisher/Spearman and the Charles approaches. The interval estimates of the dissattenuated correlation coefficient using the Fisher/Spearman and the Charles approaches produce confidence bands that are slightly wider than those around the obtained correlation coefficient and, of course, suggest the correlation between the variables without measurement error will be stronger than that observed.

Although the observed sample correlation is only .620, after adjusting for the measurement error in the two variables (i.e., r_{ii} = .70 and .80, respectively) the expected correlation is notably stronger (r = .829). Without adjusting for the attenuation of the correlation, the 95% confidence interval (the second row of the interval estimates part of the printed output) ranges from .494 to .720 In contrast, following the disattenuation the Fisher/Spearman 95% confidence interval ranges from .660 to .953. The Charles approach yields an interval that is very similar to the Fisher/Spearman interval: with the Charles approach, we can say we are 95% confident that the population correlation is between .654 to .933 if the two variables were measured with perfect reliability.

SIMULATION RESEARCH ON CONFIDENCE INTERVALS

Kromrey, Fay, and Bellara (2008) conducted a simulation study to investigate the accuracy and precision of the confidence intervals for disattenuated correlation coefficients. In this Monte Carlo study, random samples were generated under known and controlled population conditions. Samples were generated from bivariate, normal populations, measurement error was induced in each variable, and the correlation was estimated based on the sample coefficient and the disattenuated coefficient (with disattenuation based on both the known reliability (i.e., the true reliability based on the simulation parameters) and the sample estimate of the reliability). The Monte Carlo study included three factors in the design: (a) the true correlation in the population (with ρ_{XY} = .10, .30, .50, .70, and .90), (b) the reliabilities of the two variables (with ρ_{XX} = .60, .70, .80, and .90), and (c) sample size (with *N* = 25, 50, 100, 250, 500, and 1000).

For each condition investigated in the study, 10,000 samples were generated. The use of 10,000 estimates provides adequate precision for the investigation of the sampling behavior of point and interval estimates of the correlation coefficient. For example, 10,000 samples per condition provide a maximum 95% confidence interval width around an observed proportion that is $\pm$.0098 (Robey & Barcikowski, 1992). The outcomes of interest in this simulation study included both point estimates (the bias and sampling error of the observed and disattenuated correlation coefficients) and interval estimates (confidence interval coverage and width for interval estimates based on the observed coefficient, the Fisher/Spearman interval, and the Charles (2005) interval).

An overall summary of the confidence interval results is provided in Figures 1 – 4. For each condition investigated, the proportion of confidence intervals that contained the population parameter from the data generation was calculated. These proportions provide estimates of the interval coverage probabilities. Figure 1 provides a distribution of these coverage probabilities across all conditions investigated in the simulation study, when the disattenuation and the interval estimates were based on the known reliabilities of the variables. The figure presents the distributions of the coverage probabilities for confidence intervals constructed around the observed correlation (no disattenuation), and those constructed using the Fisher/Spearman approach and the Charles approach. Note the very poor coverage in many conditions for confidence intervals constructed without disattenuation. In contrast, both the Fisher/Spearman and the Charles intervals provide near nominal-level coverage (95%) across most conditions. The Fisher/Spearman interval coverage was slightly more consistent in coverage probabilities than the Charles intervals.

Figure 2 also illustrates the distribution of interval coverage probabilities across all conditions investigated in this study, but compares the disattenuation and interval estimates based on the sample reliabilities. These data, like those presented in Figure1, also reveal the distributions of coverage probabilities for both the observed correlation coefficients and the Fisher/Spearman and Charles intervals for the disattenuated correlation coefficients. The intervals for the observed coefficients still show poor coverage compared to the other two. However, in this case, the variability in the coverage probabilities between the Charles and Fisher/Spearman methods appears to be negligible.

The final two figures display the distributions of the confidence interval widths for the observed coefficients (without disattenuating), for the disattenuated coefficients using Fisher/Spearman and for the disattenuated coefficients using the Charles method. Figure 3 shows the distribution of the widths using the known reliabilities, while Figure 4 displays the bands computed using sample reliability estimates. It can be seen in both figures (in both sets of intervals) that the differences in confidence interval widths between the Fisher/Spearman and Charles methods once again appear to be negligible. It is, however, patently obvious that the concomitant confidence interval bands for the observed correlation coefficients are much narrower in width than the two disattenuated intervals.

Table 2
Sample HTML Output from Macro.

Disattenuation of Correlation Coefficients

Observed Correlation:	0.620
Reliability Estimates:	0.700
	0.800
Sample Size:	200
Disattenuated Correlation:	0.829

Interval Estimates:

	Observed Correlation		Disattenuated Correlation			
	Fisher Interval		Fisher Interval		Charles Interval	
Confidence Level	Lower	Upper	Lower	Upper	Lower	Upper
90%	0.543	0.687	0.725	0.918	0.727	0.901
95%	0.527	0.699	0.704	0.934	0.704	0.912
99%	0.494	0.720	0.660	0.963	0.654	0.933

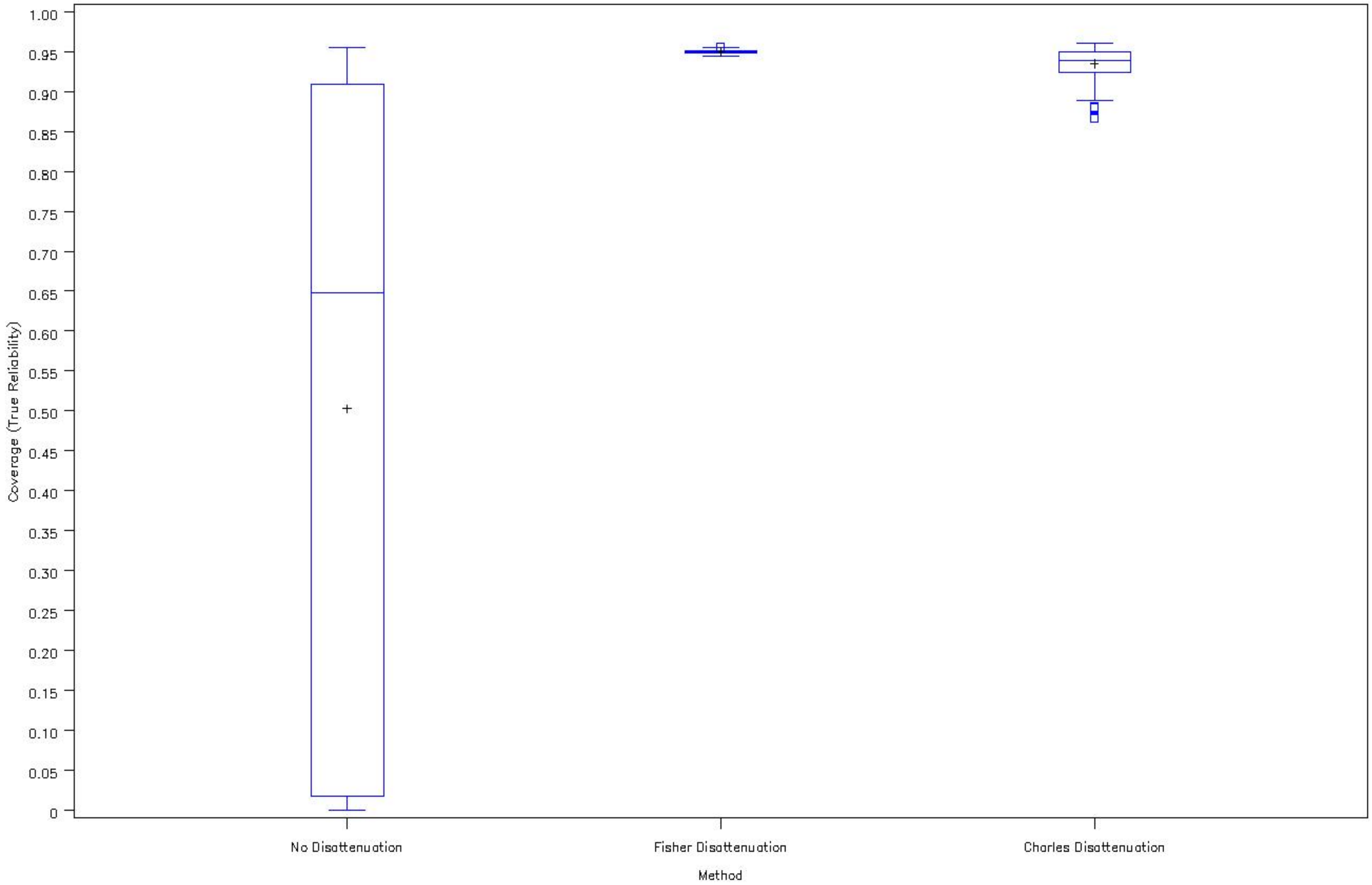

Figure 1. Distribution of Estimated Confidence Interval Coverage Using True Reliability

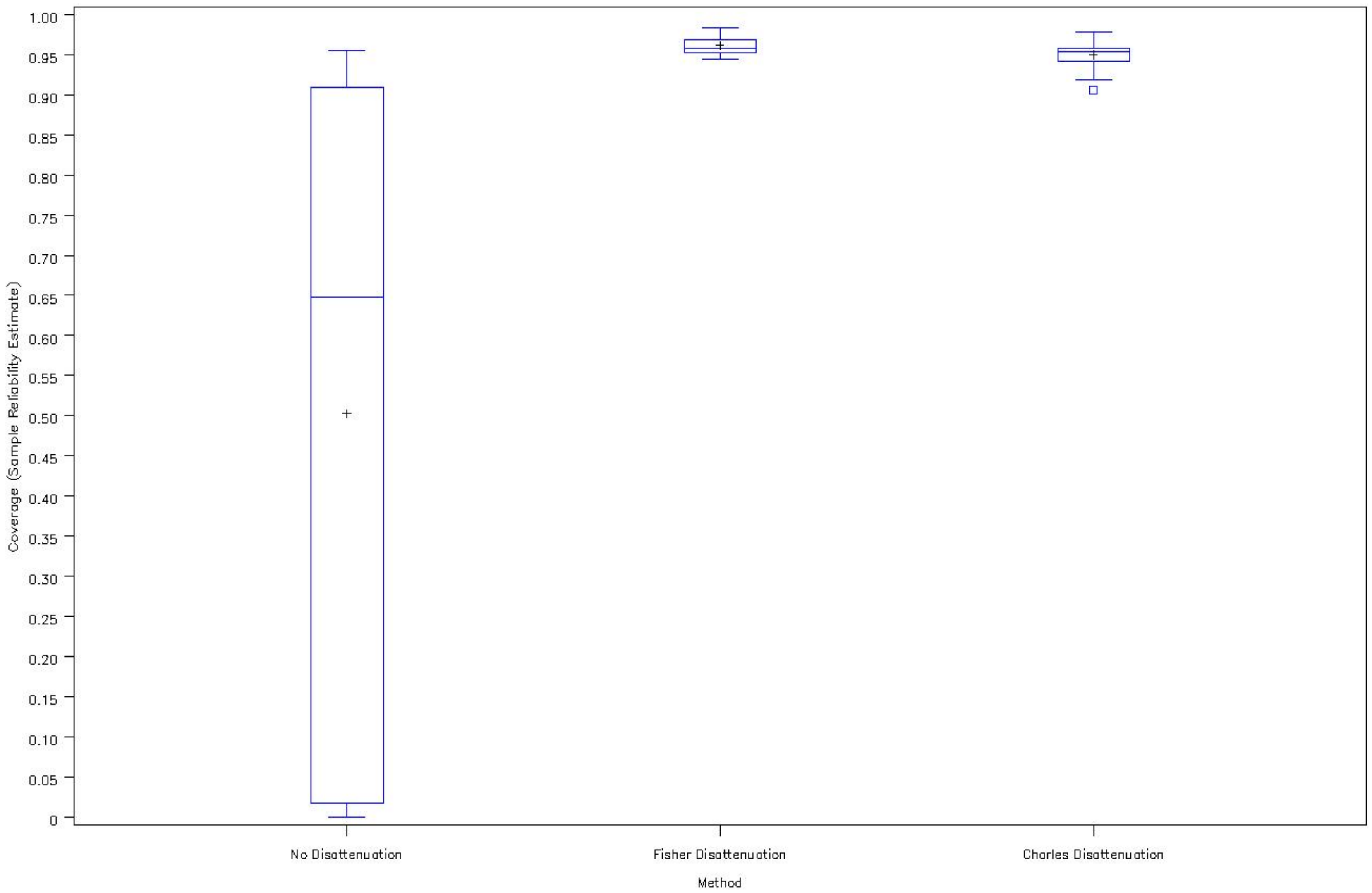

Figure 2. Distribution of Estimated Confidence Interval Coverage Using Sample Reliability

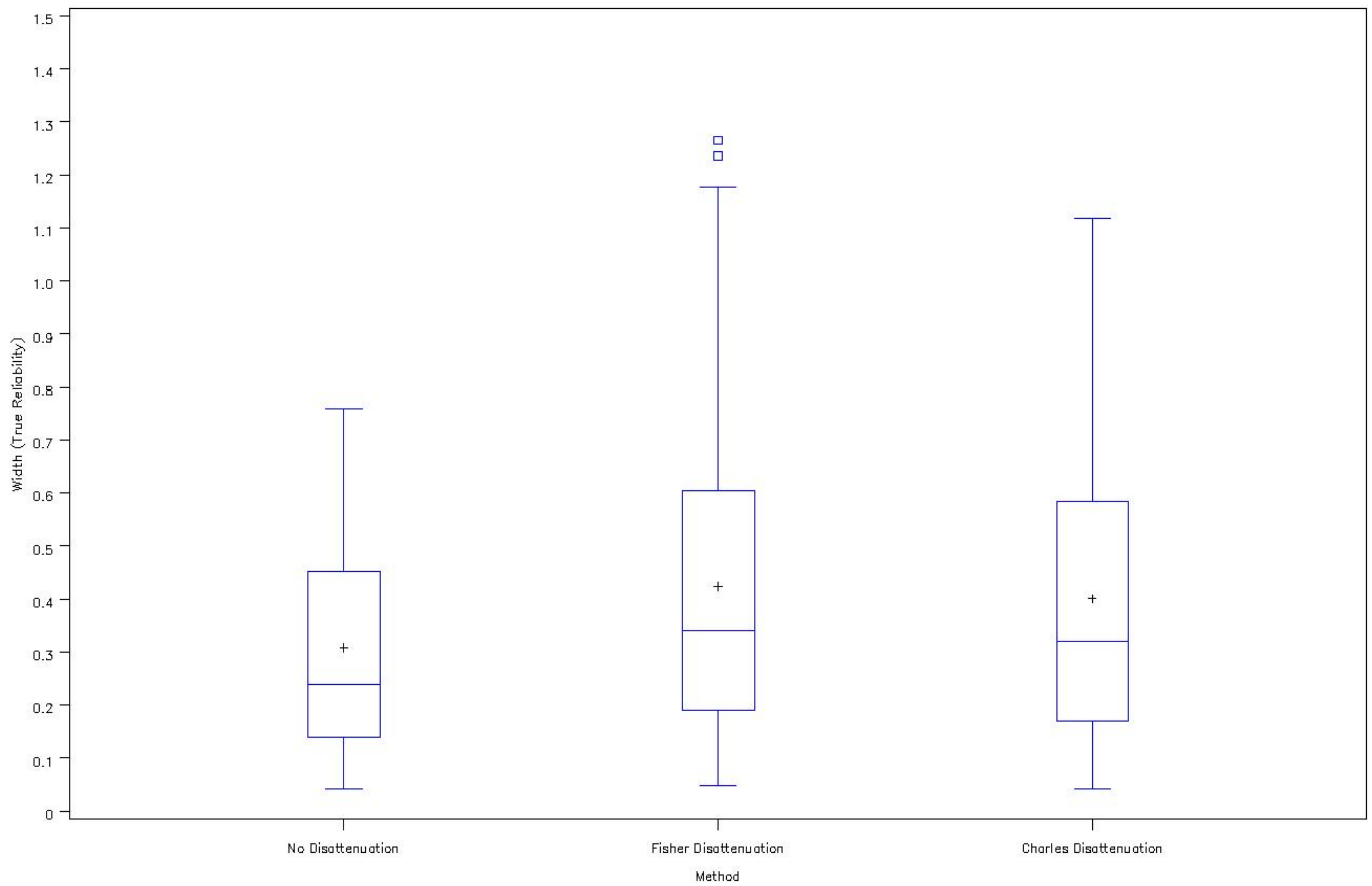

Figure 3. Distribution of Estimated Confidence Interval Width Using True Reliability

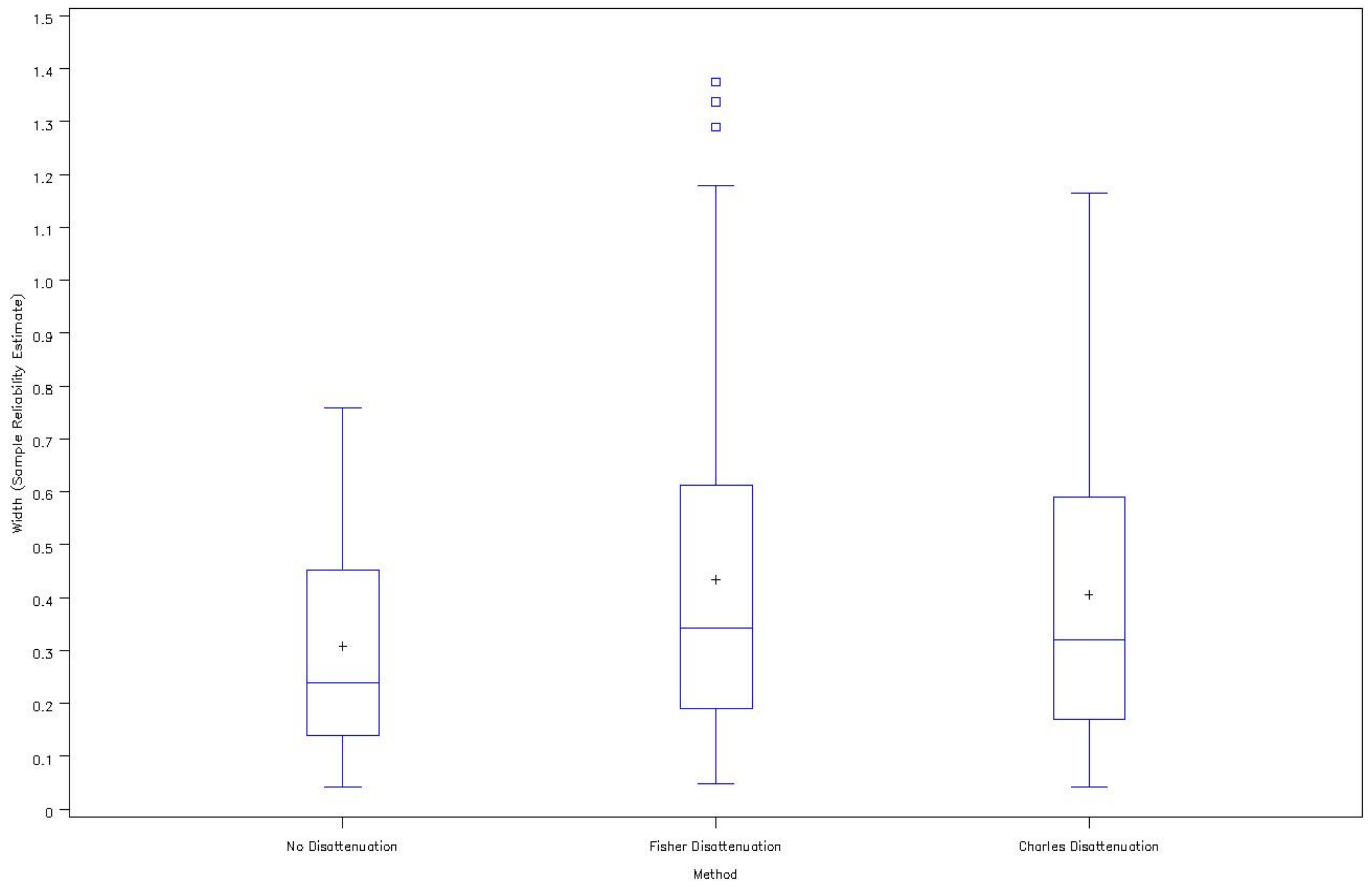

Figure 4. Distribution of Estimated Confidence Interval Width Using Sample Reliability

CONCLUSION

The potential to adjust correlation coefficients for attenuation resulting from measurement error is an important tool for researchers to consider. Empirical research has demonstrated the importance of being mindful with regard to measurement error in the interpretation of research results. That is, point estimates and interval estimates based on the observed correlation coefficient become notably less useful (with increasing statistical bias and decreasing accuracy) as the degree of measurement error in the observed variables increases. Of course, the presentation and elucidation of disattenuated correlation coefficients requires the concomitant articulation of appropriate caveats (e.g., to avoid being accused of 'interpreting what the data would have yielded if they had been psychometrically better than they really are!').

The macro ATTENUATE is provided to facilitate researchers' calculation and use of both point and interval estimates of disattenuated correlation coefficients. The macro is easy to use and is written entirely in Base SAS software, with all calculations and output provided through a SAS data step. Further, the macro may be easily modified to retain the disattenuated point estimates and the confidence interval endpoints in a SAS data set for use in further analysis or for graphical display.

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